2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Team Contest 2006/7/12 Wenzhou, China

Team: ______________________ ***Score:*** _______________

1. The teacher said, "I want to fit as large a circle as possible inside a triangle whose side lengths are 2, 2 and 2*x* for some positive real number *x*. What should the value of *x* be?" Alex said, "I think *x* should be 1." Brian said, "I think *x* should be $x=\sqrt{2}$." Colin said, "Both of you are wrong." Who was right?

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2. A triangle can be cut into two isosceles triangles. One of the angles of the original triangle is 36°. Determine all possible values of the largest angle of the original triangle.

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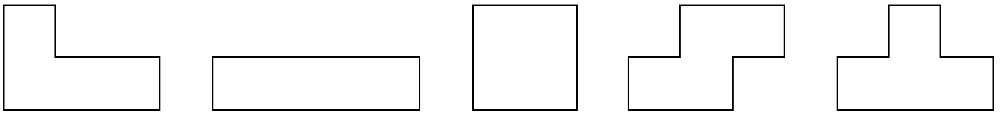
3. There are five Tetris pieces, each consisting of four unit squares joined edge to edge. Use the piece shaped like the letter L (the first one in the diagram below) and each of the other four pieces to form a shape with an axis of reflectional symmetry.



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4. A domino consists of two unit squares joined edge to edge, each with a number on it. Fifteen dominoes, numbered 11, 12, 13, 14, 15, 22, 23, 24, 25, 33, 34, 35, 44, 45 and 55, are assembled into the 5 by 6 rectangle shown in the diagram below. However, the boundary of the individual dominoes have been erased. Reconstruct them.

1	1	3	5	2	3
1	4	3	1	5	2
2	4	5	5	3	2
3	3	1	1	2	4
2	5	4	5	4	4

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5. A lucky number is a positive integer which is 19 times the sum of its digits (in base ten). Determine all the lucky numbers.

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6. Alice and Betty play the following game on an $n\times n$ board. Starting with Alice, they alternately put either 0 or 1 into any of the blank squares. When all the squares have been filled, Betty wins if the sum of all the numbers in each row is even. Otherwise, Alice wins.
   (a) Which player has a winning strategy when $n=2006$?
   (b) Answer the question in (a) for an arbitrary positive integer n**.**

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7. Prove that $1596^n + 1000^n - 270^n - 320^n$ is divisible by 2006 for all positive odd integer *n*.

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8. From the list of positive integers in increasing order, delete all multiples of 4 and all numbers 1 more than a multiple of 4. Let S_n be the sum of the first *n* terms in the sequence which remains. Compute $\left[\sqrt{S_1}\right]+\left[\sqrt{S_2}\right]+\ldots\ldots+\left[\sqrt{S_{2006}}\right]$.

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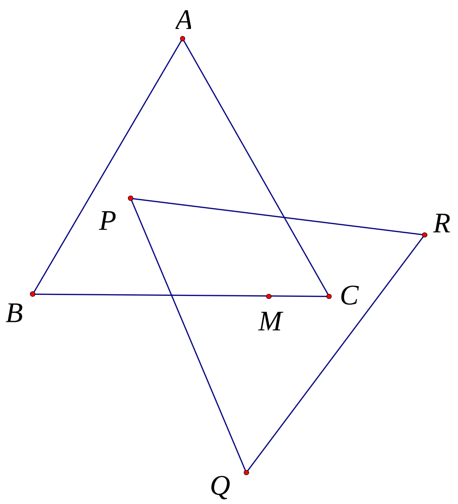
9. *ABC* and *PQR* are both equilateral triangles of area 1. The centre *M* of *PQR* lies on the perimeter of *ABC*. Determine the minimal area of the intersection of the two triangles.



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10. For a certain positive integer m, there exists a positive integer n such that mn is the square of an integer and $m-n$ is prime. Determine all such positive integers m in the range $1000 \leqq m < 2006$.